L^Hopital's Rule

Some limits can be evaluated using L^Hopital's Rule:

Indeterminate Forms: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 0^0, 1^∞, ∞^0

L'Hopital's Rule. Suppose that $f(x)$ and $g(x)$ are differential functions and $g'(x) \neq 0$ on an open interval I that contains a (except possibly for a). If

$$\lim_{x\to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x\to a} g(x) = 0$$

or

$$\lim_{x\to a} f(x) = \pm\infty \qquad \text{and} \qquad \lim_{x\to a} g(x) = \pm\infty,$$

then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

provided the right-hand-limit exists (or equals $\pm\infty$).

Note: Our goal will always be to write any indeterminate forms as one of the first two: $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

Example of L^HOPITAL'S RULE: INDETERMINATE FORM $\frac{0}{0}$ When you try and evaluate the limit, the result is $\frac{0}{0}$. So take the derivative of the numerator and denominator then evaluate.

$$\lim_{x\to 0} \frac{e^x - \cos x - 2x}{x^2 - 2x} = \lim_{x\to 0} \frac{e^x + \sin x - 2}{2x - 2} = \frac{-1}{-2} = \frac{1}{2}$$

***WARNING: YOU CANNOT USE L^HOPITAL'S FOR ALL LIMITS, JUST LIMITS IN INDETERMINATE FORM. THE ONLY INDETERMINATE FORMS ON THE AP EXAM ARE $\frac{0}{0} \, and \, \frac{\infty}{\infty}$

***IN ADDITION, SOME OF THESE LIMITS CAN BE EVALUATED DIFFERENTLY. FOR INSTANCE, EXAMPLE, #1 CAN BE FACTORED AND EXAMPLE #3 IS A SPECIAL CASE, AND EXAMPLE #4 IS AN 'EATS DC'.

Examples:

1)	2)	3)	4)
$\lim_{x\to 4} \frac{x^2 - 16}{x^2 - 5x + 4} =$	$\lim_{x\to 0} \frac{2^x - 1}{x} =$	$\lim_{x\to 0} \frac{\sin x}{x} =$	$\lim_{x\to\infty} \frac{5 + 3x^2}{4x^2 + 1} =$

ANSWERS:

1) 8/3 2) ln2 3) 0 4) 3/4